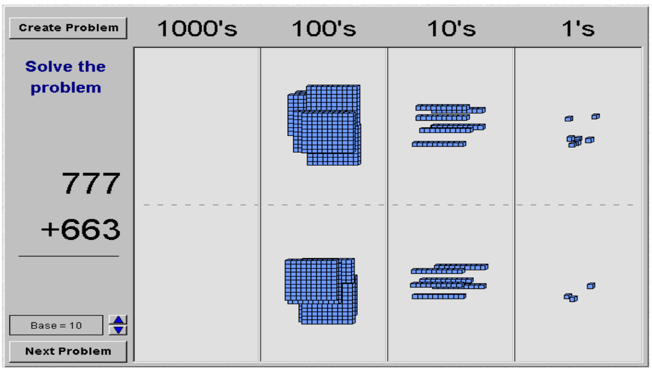
Virtual Manipulatives: Design-based Countermeasures to Selected Potential Hazards

William R. Speer, Ph.D.
Associate Dean for Academic Affairs, College of Education, Director, UNLV Center for Mathematics and Science Education, University of Nevada Las Vegas, william.speer@unlv.edu

Abstract
Virtual manipulatives are employed by both preservice and inservice teachers to enhance the instructional effectiveness of physical manipulatives and related tools by addressing limitations of access, cost, and adaptability. While research into the use of emerging technologies continues, there are several variables to consider when measuring the effects of virtual manipulative use. Research design, sampling characteristics, and the type of manipulative used may influence achievement. Variables that may influence the effectiveness of virtual manipulatives include: previous experience with computers, grade level, mathematical topic, treatment length, student attitudes toward mathematics, and computer-to-student ratio.

Introduction
One pedagogical technique commonly used in mathematics education is to provide students opportunities to actively manipulate certain aspects of the phenomenological world (Heddens, Speer & Brahier, 2009; Izydorczak, 2003; Moreno, 2005; NCTM, 2000). This technique relies on careful construction of those phenomena that exemplify the mathematical concept being conveyed. In essence, these phenomena serve as concrete analogies of mathematical concepts and, in the language of mathematics education, are said to *model*[1] those concepts. Pedagogical tools specifically designed for this type of active manipulation are called "manipulatives." With the advent of digital technology, this basic idea of manipulatives has been extended to the computer-based manipulatives or "virtual manipulatives" (Schackow, 2007; Tversky & Morrison, 2002).
This paper identifies and discusses 1) some potentially detrimental effects that the use of virtual manipulatives may have on mathematical learning, and 2) possible ways to address these effects. The issues identified in this paper are based on observations of students interacting with a number of virtual manipulatives found on the Internet, namely, at the National Library of Virtual Manipulatives for Interactive Mathematics site. This paper reports on test cases and anecdotal evidence that lack the necessary basis for definitive conclusions and, consequently, is presented as a preliminary study upon which further, more rigorous, investigation may be formulated.

Methodology
This paper is based on two different types of observations. One type was based on a series of one-hour sessions with 4th grade students. Sessions consisted of a series of addition exercises each first attempted using the virtual manipulative and then using the physical manipulative (Mousavi, Low, & Sweller, 1995; Schnotz, 2005).
This screen shot shows an exercise with the virtual Base-10 Blocks manipulative:

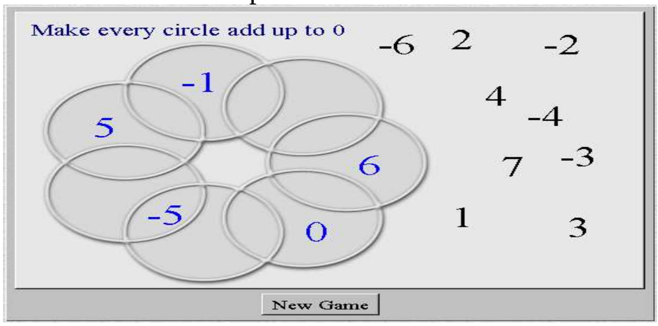


Here, the object of the manipulative is to 1) aggregate 10 pieces in one unit into a single piece in the next higher unit, and 2) place each piece in the columns to which they belong.
Additional observations were based on three "computer-lab" sessions of three different middle school mathematics classes. Most of the students worked individually, each with a dedicated computer. However, due to a limited number of computers, a small minority of students worked in groups of two. Also, on occasion, especially when students appeared to be "stuck" in a particular scenario, the observer interjected with questions and suggestions to the students. Each lab session began with the students exploring the

---

[1] It is curious that the notion of *modeling* in mathematics education is a mirror image of that found in science. In physics, for example, mathematical concepts are used to *model* physical phenomenon. Here, the object of study is the phenomenological world and mathematics is a language used to describe that world. In mathematics education, on the other hand, the object of study is mathematics itself and the phenomenological world is used to *model* mathematical concepts.

Circle-0 virtual manipulative.



The object of Circle-0 is to place all the numbers (using the drag-and-drop technique with the mouse) within the circles so as to make each circle to "add up to 0".

Potential Issues

This section of the paper identifies 4 different ways virtual manipulatives may be counter-productive to learning. Each is discussed in terms of: characterization of the potential issue; observations that support this characterization; ramifications of this issue for effective and efficient learning; potential design solutions to address these issues; and, procedures devoid of concepts.

Under certain circumstances, manipulations may not be accompanied by their intended conceptual counterpart. Students may acquire procedural expertise needed to successfully complete the manipulatives without internalizing the concepts that the manipulatives were designed to model (Atkinson, 2002; Izydorczak, 2003).

An observation that prompted this issue was with a student's interaction with Base-10 Blocks. With virtual Base-10 Blocks, the procedure for combining 10 pieces in one unit required surrounding 10 *or more* pieces with a bounding rectangle with the mouse. Once those pieces were successfully bound, the computer automatically transformed 10 of those pieces into a single piece of the next higher unit. It became clear that the student became focused on the operation of surrounding all of the pieces in each unit with the bounding rectangle. From the student's perspective, this was a reasonable strategy for the mastery of that skill - irrespective of any concepts that may be associated with it - was what was necessary to complete the exercise.

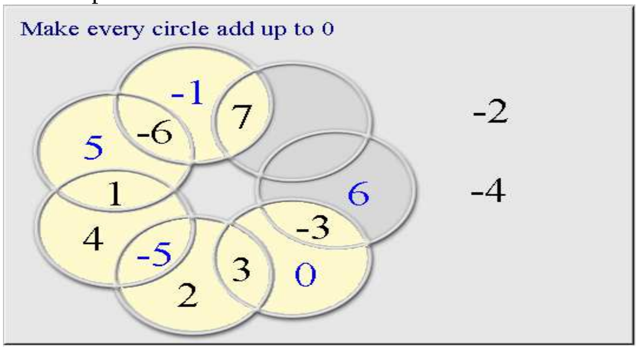
The fact that the student had difficulty replicating his solution with the physical Base-10 Blocks lent additional credence that there was a chasm between his understanding of the procedural requirements of the manipulative and their corresponding meaning in the number system. With a clear understanding of the relationship between the two, one would expect that changing the medium of the manipulatives - from virtual to physical - would have had less impact than was observed (Drickey, 2001).

Clearly, when students "play the game" devoid of the concepts the manipulative is designed to demonstrate, the effectiveness of the exercise in meeting the intended pedagogical goals is likely limited (Atkinson, 2002). However, this potential for procedural expertise devoid of conceptual understanding seems to be an inherent vulnerability of manipulatives in general (Thompson, 1992). Manipulatives are, in essence, phenomenological analogies for concepts and, as such, always carry the possibility of being misconstrued. Of course, the task of educators is to limit the likelihood of those misconceptions and to resolve them when they do occur.

What, then, are some potential strategies to 1) limit the likelihood of conceptually empty procedural manipulations, and 2) resolve them when they occur? With respect to the virtual Base-10 Block example cited, the virtual manipulative may be designed so that: the student must explicitly collect exactly 10 pieces of one unit rather than simply surround 10 or more pieces within a bounding rectangle; the student must explicitly request (say, using a button) to convert the 10 pieces to a single piece of the higher unit; and/or the student must replicate the manipulation with symbolic operations. In general, by requiring of the student greater responsibility of the manipulations (as opposed to automated manipulation by the system) and corresponding symbolic operations, the likelihood that the student will make the conceptual connection with procedural operations may be increased (Pass, Renkl, & Sweller, 2003).

Local Minimum

Some students seem to get "stuck" in a local minimum of the search space. As a result, those unwilling or unable to backtrack (i.e., give up some of the gains seemingly achieved) were unable to complete the problem (Schnotz & Bannert, 2003). The level of challenge represented by these local minimums may be counter-productive to some students. This issue was observed particularly with the Circle-0 virtual manipulative.



In this example, 5 out of 7 circles have met the criteria, i.e. they "add up to 0". There are two numbers left (-2 and -4) which must be placed in the two remaining spots (between 7 and 6) so that the remaining two circles also add up to 0. It is easy to see that neither of the two combinations of placing the remaining numbers produces the solution. Therefore, in order to reach the solution where all the circles add up to 0, the partial solution generated so far must be sacrificed.

The question remains as to whether such perplexity is conducive or counter-productive to learning. It seems that, in general, the answer depends on the student (Schnotz & Rasch, 2005). The detrimental effect of too much challenge seems particularly relevant in the area of self-efficacy. Specifically within the context of Circle-0, the level of challenge posed by a solution that seems so close and yet so far is almost always inappropriate for the students for which the manipulative is target, i.e. students learning to add single digit integers.

One way to control the potential level of local minimum is through additional constraints. In the case of Circle-0, additional starting numbers may be added so as to remove the possibility of deep local minimums. Given the size of the search space and the computational speed of even modest computers, it should be feasible to check all possible points on the search space so as to ensure maximum level of difficulty in terms of local minimums.

Disengagement

Some students seemed to be disengaged with the problem at hand and repeatedly hit either the "Hint" or the "Reset" button without demonstrating any attempts to actually solve the problem presented. To the observer, it was as if, once they had developed the pattern, they were stuck in a mental mode of simply pressing those buttons. Needless to say, such a lack of engagement with the problem is counter-productive to learning.

The problem of disengagement seems not so much a problem with virtual manipulatives, but a manifestation of a more general issue in learning (Mayer & Chandler, 2001). A student's inability or unwillingness to effectively engage a problem likely points to more fundamental issues in learning and may require higher levels of intervention. Therefore, it seems unlikely that this type of disengagement would be eliminated by simple design changes in the manipulatives. At least in principle, however, it may be possible to enable the virtual manipulative system to automatically 1) detect certain patterns of use (or rather misuse) of the program and 2) to provide some type of interjection (or notification to human instructor).

From a technological perspective, the level of computational sophistication needed for such functionality is qualitatively different than what is found in implementations of virtual manipulatives at the National Library of Virtual Manipulatives for Interactive Mathematics site. This type of functionality - automated diagnosis of student performance and feedback based on that diagnosis - has yet to be effectively demonstrated in much of educational technology. It is the author's view that the development of this class of functionality will become a focus of research.

Rule Confusion

Some students seemed to not understand what was being asked of them even after they spent some time reading the accompanying instructions. One group of students seemed in a state of bewilderment with the Circle-0 manipulative. However, once they were shown how to fill in one or two circles, they immediately and quickly progressed with the remaining circles. The amount of cognitive and emotional support seemingly required during these periods of confusion is likely counter-productive. Certainly, from the learner's point of view, this can be both unpleasant as well as counterproductive.

One way to increase the understandability of the instruction may be to simplify the language of the instruction. The online instruction for the Circle-0 manipulative, for example, is shown:

Circle 0

This virtual manipulative poses the problem:

Position the 14 numbers inside of the circles and intersections of circles in such a way that the three numbers in each of the circles sums (adds up) to zero.

Work on the problem by clicking on and dragging numbers to locations inside circles or intersections of circles.

When the three numbers in a circle add up to zero, the circle will change color.

It is sometimes unclear for whom the instructions are intended. Using a language more suitable to younger students may increase the likelihood of being understood. An even more intuitive and effective way (albeit, more difficult and costly) to convey the instructions for using the manipulatives may be through animated demonstration of its use. Animation may be effective in communicating not only basic instruction but also different strategies for tackling a problem (Atkinson, 2002).

Conclusion

Perhaps, it should be axiomatic that every technology has its limitations and, therefore, can be misused. This paper discussed some the potential inherent and design limitations of virtual manipulatives and how these limitation may be addressed. One of the conclusions based on the observations cited in this paper may be that appropriate supervision is needed to maximize/minimize the potential benefit/detriment of virtual manipulatives. It is the view of this author that one of the major areas of research in educational technology is the development of assistive mechanisms to effectively and efficiently support this type of interaction.

References

Atkinson, R. (2002). Optimizing Learning From Examples Using Animated Pedagogical Agents. *Journal of Educational Psychology*, 94, 416–427.

Drickey, N.A. (2001). A comparison of virtual and physical manipulatives in teaching visualization and spatial reasoning to middle school mathematics students. Unpublished doctoral dissertation, Utah State University, 2001.

Heddens, J.W., Speer, W.R., & Brahier, D.J., (2009). Today's mathematics: Concepts, classroom methods, and instructional activities. 12th ed., John Wiley & Sons, New York.

Izydorczak, A. (2003). A Study of Virtual Manipulatives for Elementary Mathematics. Unpublished doctoral dissertation, State University of New York - Buffalo, 2003.

Mayer, R. E., & Chandler, P. (2001). When learning is just a click away: Does simple user interaction foster deeper understanding of multimedia messages? *Journal of Educational Psychology*, 93, 393-397.

Mayer, R. E., & Moreno, R. (1998). A split-attention effect in multimedia learning: evidence for dual processing systems in working memory. *Journal of Educational Psychology*, *90*, 312-320.

Moreno, R. (2005). Multimedia learning with animated pedagogical agents. In R. Mayer (Ed.), *Cambridge Handbook of Multimedia Learning*. Cambridge University Press.

Mousavi, S. Y., Low, R., & Sweller, J. (1995). Reducing cognitive load by mixing auditory and visual presentation modes. *Journal of Educational Psychology*, *87*, 319-334.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.

Pass, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design - recent developments. *Educational Psychologist*, 38, 1-4.

Schackow, J.B. (2007) Using Virtual Manipulatives to Model Computation with Fractions. ON-Math, 5(1), 1-5.

Schnotz, W. (2005). An integrated model of text and picture comprehension. In R. Mayer (Ed.), *Cambridge Handbook of Multimedia Learning*. Cambridge University Press.

Schnotz, W. & Bannert, M. (2003). Construction and interference in learning from multiple representation. *Learning and Instruction*, 13, 141-156.

Schnotz, W. & Rasch, T. (2005). Enabling, facilitating, and inhibiting effects of animation in multimedia learning: Why reduction of cognitive load can have negative results on learning. *Educational Technology Research and Development*, *53*, 47-58.

Thompson, P. W. (1992). Notations, conventions, and constraints: Contributions to effective uses of concrete materials in elementary mathematics. Journal for research in mathematics education, 23, 123-147.

Tversky, B. & Morrison, J. B. (2002). Animation: can it facilitate? *International Journal of Human-Computer Studies*, 57, 247-262.